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Extended Electron and Nonlocal Electromagnetic Interaction: The Perturbation Theory

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The nonlocal interaction between electrons and electromagnetic fields is considered. It is shown that different contraction forms of interacting fields are equivalent to different nonlocal theories where nonlocality is connected to either the photon field or the electron field, or to both these fields simultaneously. The nonlocal theory where the electron carries nonlocality is studied in detail. The gauge invariance of this model is achieved by using the *d*-operation applying the perturbation theory. Primitive Feynman diagrams of the nonlocal theory are investigated and a restriction on the "size" l of the electron is obtained. From low-energy experimental data from tests of local quantum electrodynamics it follows that $l \leqq 10^{-15}$ cm.

1. INTRODUCTION

The concept of extended (or spread-out) particles plays an important role in the construction of a unified field theory of elementary particle interactions. One of the best examples is the string field theory (Green *et al.*, 1987), which underlies the physics of two-dimensional field theory. In this paper we use the Efimov (1977) approach to the description of the nonlocal interaction of quantized fields. This theory is phenomenological, where nonlocal (extended) objects are constructed by using a form factor of the theory; for example, the wave function of the nonlocal electron takes the form

$$\Psi(x) = \int d^4y\, K_l(y)\psi(x - y) \tag{1.1}$$

where $\psi(x)$ is the local wavefunction of the electron; $K_l(y)$ is the form factor connected with the "structural size" l of the electron and satisfies the normalization condition

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$$\int d^4y\, K_l(y) = 1 \tag{1.2}$$

in Euclidean space. Its Fourier transform is an entire analytic function in momentum space, for example,

$$\tilde{K}(p^2l^2) = \exp\left(-\frac{p^2 + m^2}{4l^2}\right) \tag{1.3}$$

where

$$p^2 = p_4^2 + \mathbf{p}^2$$

We know that the local field of the electron $\psi(x)$ satisfies the Dirac equation

$$\left(i\gamma_\mu \frac{\partial}{\partial x_\mu} + m\right)\psi(x) = 0 \tag{1.4}$$

Then the equation of motion for the extended electron field (1.1) is

$$\int d^4y\, K_l(y)\left(i\gamma_\mu \frac{\partial}{\partial x_\mu} + m\right)\psi(x - y) = 0 \tag{1.5}$$

In a previous paper (Namsrai and Njamtseren, 1994) we have studied gauge invariance of the given scheme by using the d-operation (Kroll, 1966) [for details see Namsrai (1986)] in the momentum representation. This work is devoted to the construction of the perturbation theory where only the electron carries nonlocality, and investigates primitive Feynman diagrams, which allows us to obtain a restriction on the "size" of the electron from low-energy experimental data.

2. GAUGE INVARIANCE AND INTRODUCING INTERACTION OF ELECTROMAGNETIC FIELD

2.1. The Local Case

It is well known that the Dirac equation (1.4) is invariant under global gauge transformation

$$\psi(x) \Rightarrow \psi'(x) = e^{i\alpha}\psi \tag{2.1}$$

where α is a constant parameter. If instead of (2.1) we consider the local gauge transformation

$$\psi'(x) = S^{-1}\psi(x), \qquad S^{-1} = \exp[ief(x)] \tag{2.2}$$

[where $f(x)$ depends on variable x], then the Dirac equation (1.4) is not invariant under the transformation (2.2). In order to compensate for the term breaking gauge invariance in the Dirac equation, we should introduce a gauge field $A_\mu(x)$ into it by means of the following formal procedure:

$$\frac{\partial}{\partial x_\mu} \Rightarrow \frac{\partial}{\partial x_\mu} - ieA_\mu(x)$$

Then the Dirac equation (1.4) takes the form

$$\Lambda(x) = i\gamma_\mu \frac{\partial\psi}{\partial x_\mu} + eA_\mu(x)\gamma_\mu\psi(x) + m\psi(x) = 0 \tag{2.3}$$

This equation preserves its form (i.e., invariance holds)

$$S\Lambda'(x) = \Lambda(x) \tag{2.4}$$

where

$$S = \exp[-ief(x)], \qquad \psi'(x) = \exp[-ief(x)]\psi(x) \tag{2.5}$$

$$A'_\mu(x) = A_\mu(x) + \frac{\partial f(x)}{\partial x_\mu} \tag{2.6}$$

In our case, $A_\mu(x)$ is the electromagnetic field carrying spin 1.

2.2. The Nonlocal Case

For the nonlocal field (1.1), a gauge-invariant equation similar to (2.4) yields the form

$$\int d^4y\, K_f(y)S\Lambda'(x-y) = \int d^4y\, K(y)S\Lambda(x-y) = 0 \tag{2.7}$$

where

$$\Lambda'(x-y) = i\gamma_\mu \frac{\partial\psi'(x-y)}{\partial x_\mu} + eA'_\mu(x-y)\gamma_\mu\psi'(x-y) + m\psi'(x-y)$$

In the given case $S = \exp\{-ief(x-y)\}$.

The integral equation (2.7) allows us to introduce a nonlocal interaction Lagrangian instead of the local one,

$$L_{\text{int}}(x) = e\bar{\psi}(x)\gamma_\mu\psi(x)A_\mu(x) \tag{2.8}$$

Here we consider four different forms of the interaction Lagrangian.

3. DIFFERENT FORMS OF THE INTERACTION LAGRANGIAN

We use the following possible contraction terms:

$$L^1_{\text{in}}(x) = e\int d^4y\, K_l(y)\bar{\psi}(x-y)\gamma_\mu\psi(x-y)A_\mu(x-y) \tag{3.1}$$

$$\begin{aligned} L^2_{\text{in}}(x) = e\iint d^4y_1\, d^4y_2\, K_l(y_1)K_l(y_2) \\ \times \bar{\psi}(x-y_1-y_2)\gamma_\mu\psi(x-y_1-y_2)A_\mu(x-y_1) \end{aligned} \tag{3.2}$$

$$\begin{aligned} L^3_{\text{in}}(x) = e\iint d^4y_1\, d^4y_2\, K_l(y_1)K_l(y_2) \\ \times \bar{\psi}(x-y_1)\gamma_\mu\psi(x-y_2)A_\mu(x-y_2) \end{aligned} \tag{3.3}$$

$$\begin{aligned} L^4_{\text{in}}(x) &= e\iiint d^4y_1\, d^4y_2\, d^4y_3\, K_l(y_1)K_l(y_2)K_l(y_3) \\ &\quad \times \bar{\psi}(x-y_1)\gamma_\mu\psi(x-y_2)A_\mu(x-y_1-y_2-y_3) \\ &= e\iint d^4y_1\, d^4y_2\, K_l(y_1)K_l(y_2) \\ &\quad \times \bar{\psi}(x-y_1)\gamma_\mu\psi(x-y_2)A^{\text{nonlocal}}_\mu(x-y_1-y_2) \end{aligned} \tag{3.4}$$

where

$$A^{\text{nonlocal}}_\mu(x-z) = \int d^4y_3\, K_l(y_3)A_\mu(x-z-y_3)$$

By using the perturbation theory in the momentum representation in the Euclidean metric one can easily show that:

1. The interaction Lagrangian (3.1) is equivalent to the local one (2.8), i.e., it describes the local theory.

2. The Lagrangian (3.2) leads to the nonlocal theory where only the photon field is responsible for nonlocality

$$L^2_{\text{in}}(x) = e :\bar{\psi}(x)\gamma_\mu\psi(x)A^{\text{nonlocal}}_\mu(x): \tag{3.5}$$

where

$$A^{\text{nonlocal}}_\mu(x) = \int d^4y\, K_l(y)A_\mu(x-y)$$

and its propagator has the form

$$D^{l}_{\mu\nu}(x-y) = \langle 0 | T[A^{\text{nonlocal}}_{\mu}(x) A^{\text{nonlocal}}_{\nu}(y)] | 0 \rangle$$

$$= \frac{1}{(2\pi)^4 i} g_{\mu\nu} \int d^4p \, e^{-ip(x-y)} \frac{1}{p^2 + i\epsilon} \tilde{K}^2(-p^2 l^2) \quad (3.6)$$

3. The interaction Lagrangian (3.3) deals with the nonlocal theory where nonlocality is connected with the electron field:

$$L^3_{\text{in}}(x) = e :\overline{\Psi}(x) \gamma_\mu \Psi(x) A_\mu(x): \quad (3.7)$$

where $\Psi(x)$ is the nonlocal electron field defined by the expression (1.1) and its propagator has the form

$$S_l(x-y) = \langle 0 | T[\Psi(x) \overline{\Psi}(y)] | 0 \rangle$$

$$= \frac{1}{(2\pi)^4 i} \int d^4p \, e^{-ip(x-y)} \frac{\tilde{K}^2(-p^2 l^2)}{m^2 - p^2 - i\epsilon} (m + \hat{p}) \quad (3.8)$$

4. Finally, the Lagrangian (3.4) describes the essential nonlocal theory where both the photon and electron fields carry the nonlocality property simultaneously. Here the interaction Lagrangian is

$$L^4_{\text{in}}(x) = e :\overline{\Psi}(x) \gamma_\mu \Psi(x) A^{\text{nonlocal}}_{\mu}(x): \quad (3.9)$$

In this case, matrix elements of the S-matrix are constructed by using the nonlocal propagator of the photon (3.6) and electron (3.8), respectively.

In this paper, we consider the nonlocal theory which is described by the Lagrangian (3.3).

4. EXTENDED ELECTRON AND ITS *S*-MATRIX THEORY

In our scheme, a spread-out (or extended) electron is described by the wavefunction (1.1) and its nonlocal propagator is (3.8). The S-matrix for a such theory is constructed analogously to Efimov's nonlocal theory:

$$S = T^{\delta}_{d} \exp\left\{ \int d^4x \, L^3_{\text{in}}(x) \right\} \quad (4.1)$$

where the symbol T^{δ}_{d} is the so-called Wick T-product or T^*-operation (for example, see Bogolubov and Shirkov, 1980) and δ, d correspond to some intermediate regularization procedure (Efimov, 1977) and d-operation (Namsrai, 1986) which make all matrix elements of the perturbation theory finite and gauge invariant.

In order to construct the perturbation series for the S-matrix (4.1) by prescription of the usual local theory, it is necessary to change (in the Feynman diagrams)

$$\Delta_{\mu\nu}(x-y) \Rightarrow \Delta_{\mu\nu}(x-y) \qquad \text{(unchanged photon propagator)}$$

$$S(x-y) \Rightarrow S_1(x-y)$$

in accordance with formula (3.8), and to use the generalized vertex (at the external photon lines)

$$\gamma_\mu \Rightarrow U_{1\mu}(k, q) = -d_\mu(k) S_1^{-1}(\hat{q}) \tag{4.2}$$

where the actions of the d-operation are [for details, see Kroll (1966) and Namsrai (1986)]

1. $d_\mu(k)\hat{q} = [(\hat{q} + \hat{k}) - \hat{q}] \dfrac{k\gamma_\mu}{k^2} = \gamma_\mu$

2. $k_\mu d_\mu(k)\hat{q} = \hat{k}$

3. $d_\mu(k)V(-q^2l^2) = [V(-(q+k)^2l^2) - V(q^2l^2)] \dfrac{\hat{k}\gamma_\mu}{k^2}$ (4.3)

4. $d_\mu(k)V^{-1}(-q^2l^2) = -V^{-1}(-(q+k)^2l^2)[d_\mu(k)V(-q^2l^2)]V^{-1}(-q^2l^2)$

In our case $V(-q^2l^2) = \tilde{K}^2(-q^2l^2)]$

5. $d_\mu(k)S_1(q) = S_1(\hat{q} + \hat{k})U_{1\mu}(k, q)S_1(\hat{q})$ (4.4)

where $U_{1\mu}(k, q)$ is defined by (4.2).

5. THE PRIMITIVE FEYNMAN DIAGRAMS

Let us consider the Feynman diagrams of Fig. 1 in the nonlocal theory.

5.1. The Vacuum Polarization Diagrams

The matrix element corresponding to this diagram (Fig. 1a) has the form (Namsrai, 1986)

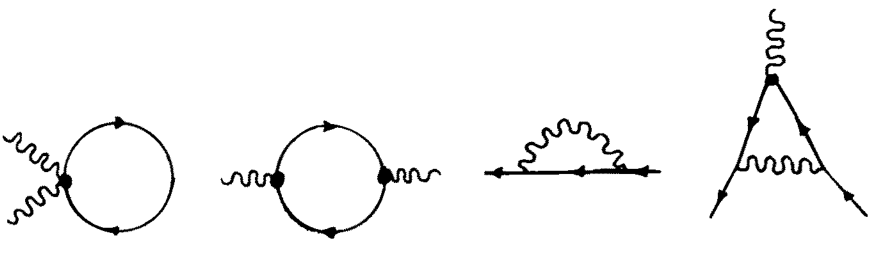
$$\Pi^1_{\mu\nu}(k_1, k_2) = \frac{e^2}{(2\pi)^4} \frac{1}{2} d^4q \; V(-q^2l^2) Sp\{\tilde{\Gamma}_{\mu\nu}(q, k_1, k_2) S_1(\hat{q})\} \tag{5.1}$$

where

$$k_1 + k_2 = 0$$

$$S_1(\hat{q}_2)\tilde{\Gamma}_{\mu\nu}(q, k_1, k_2)S_1(q) = (-1)^2 d_\mu(k_1) d_\nu(k_2) S_1(\hat{q}) \tag{5.2}$$

$$(q_2 = q + k_1 + k_2 = q)$$



a) b) c)

Fig. 1. The primitive Feynman diagrams in the nonlocal theory.

This diagram was investigated in Namsrai (1986) and has the gauge-invariant form

$$\Pi_{\mu\nu}(k) = \frac{e^2}{2\pi^2}(k_\mu k_\nu - k^2 g_{\mu\nu})\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta+i\infty} d\xi \frac{v(\xi)}{\sin\pi\xi}(m^2l^2)^\xi \frac{\Gamma(-\xi)}{\Gamma(1-\xi)}$$
$$\times \int_0^1 dx\, x(1-x)^{1-\xi}\left[1 - \frac{k^2}{m^2}x(1-x)\right]^\xi \qquad (0 < \beta < 1) \quad (5.3)$$

where we have used the form factor (1.3) and its Mellin representation

$$V(q_E^2 l^2) = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} \frac{d\xi}{\sin\pi\xi}\frac{1}{\Gamma(1+\xi)} 2^{-2\xi} l^{2\xi}[m^2 + q_E^2]^\xi \qquad (5.4)$$

5.2. The Self-Energy Diagram

The matrix element for this diagram (Fig. 1b) is

$$\tilde{\Sigma}(p) = \frac{e^2}{(2\pi)^4}\int d^4 q_E \frac{1}{(p_E - q_E)^2} V(q_E^2 l^2)\gamma_\mu^{(E)} \frac{m - \hat{q}_E}{m^2 + q_E^2}\gamma_\mu^{(E)} \qquad (5.5)$$

where we have used the Euclidean metric:

$$p = (p_4, \mathbf{p}), \qquad p_E^2 = p_4^2 + \mathbf{p}^2, \qquad p_4 = -ip_0$$

$$\hat{p}_E = (p_E\gamma^E) = p_4\gamma_4 + \mathbf{p}\boldsymbol{\gamma} = -p_0\gamma_0 + \mathbf{p}\boldsymbol{\gamma} = -(p\gamma) = -\hat{p}$$

$$\gamma_\mu^{(E)}\gamma_\nu^{(E)} + \gamma_\nu^{(E)}\gamma_\mu^{(E)} = -2\delta_{\mu\nu} \qquad (\delta_{11} = \delta_{22} = \delta_{33} = \delta_{44} = 1)$$

$$\gamma_\mu^{(E)}\hat{p}_E\gamma_\mu^{(E)} = 2\hat{p}_E \qquad \gamma_\mu^{(E)}\gamma_\mu^{(E)} = -4$$

After some elementary calculations and passing to the pseudo-Euclidean metric, one gets

$$\tilde{\Sigma}(p) = \frac{e^2}{8\pi}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{v(\xi)}{(\sin \pi\xi)^2}\frac{(m^2 l^2)^\xi}{\Gamma(1+\xi)} F(\xi, p)$$

$$F(\xi, p) = \frac{1}{\Gamma(1-\xi)}\int_0^1 dx \left(1 - \frac{p^2}{m^2}x\right)^\xi (2m - \hat{p}x) \quad (5.6)$$

where

$$v(\xi) = 2^{-2\xi}\frac{1}{\Gamma(1+\xi)}$$

in accordance with (5.4).

5.3. The Vertex Diagram

In the pseudo-Euclidean space, the vertex function corresponding to this diagram (Fig. 1c) has the form

$$\Gamma_\mu(p, q) = -i^{-1}e^2(2\pi)^{-4}\int d^4k\, \Delta(-(p-k)^2)\gamma_\nu d_\mu(q)S_l(\hat{k})\gamma_\nu \quad (5.7)$$

where by definition

$$d_\mu(q)S_1(k) = \frac{1}{m - \hat{k} - \hat{q}} V(-k^2l^2)\frac{1}{m-\hat{k}} + \frac{1}{m - \hat{k} - \hat{q}}$$
$$\times [V(-(q+k)^2l^2) - V(-k^2l^2)]\frac{\gamma_\mu \hat{q}}{q^2} \quad (5.8)$$

For calculational purposes, the following identity is useful:

$$V(-(q+k)^2l^2) - V(-k^2l^2)$$
$$= -(q^2 + 2(kq))\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{v(\xi)}{\sin \pi\xi}\xi l^{2\xi}$$
$$\times \int_0^1 dx\, [m^2 - k^2 - 2x(kq) - q^2x]^{\xi-1} \quad (5.9)$$

After standard but tedious calculations we have in the mass-shell $p^2 = p'^2 = m^2$

$$\bar{u}(p')\Gamma_\mu(p', p)u(p) = \bar{u}(p')\Lambda_\mu(q)u(p) \tag{5.10}$$

Here

$$\Lambda_\mu(q) = \gamma_\mu F_1(q^2) + \frac{i}{2m}\sigma_{\mu\nu}q_\nu F_2(q^2) \tag{5.11}$$

$$F_j(q^2) = -\frac{e^2}{8\pi}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{v(\xi)}{(\sin \pi\xi)^2}\frac{(m^2l^2)^\xi}{\Gamma(1+\xi)} f_j(\xi, q^2) \tag{5.12}$$

$$\begin{aligned} f_j(x, q^2) &= \frac{1}{\Gamma(1-\xi)}\iiint_0^1 d\alpha\, d\beta\, d\gamma\, \gamma^{-\xi}\delta(1-\alpha-\beta-\gamma) \\ &\times \left\{\frac{g_j(\alpha, \beta, \gamma, q^2)}{[\lambda\alpha + (1-\alpha)^2 - \beta\gamma q^2/m^2]^{1-\xi}}\right. \\ &+ \xi\int_0^1 dx \\ &\times \left.\frac{G_j(\alpha, \beta, \gamma, x; q^2)}{[\lambda\alpha + (1-\alpha)^2 - \beta\gamma q^2/m^2 - (q^2/m^2)(x\gamma - x^2\gamma^2 - 2x\gamma(\beta + \alpha/2))]^{1-\xi}}\right\} \end{aligned} \tag{5.13}$$

where

$$\begin{aligned} g_1 &= [(1-\alpha)^2(1-\xi) + 2\alpha\xi] - [\beta\gamma + \xi(\alpha+\beta)(\alpha+\gamma)]\frac{q^2}{m^2} \\ g_2 &= 2\xi\alpha(1-\alpha) \\ G_1 &= (1-\alpha)^2 - \left[\beta\gamma + \gamma x(1-\gamma x) - 2x\gamma\left(\beta + \frac{\alpha}{2}\right)\right. \\ &\quad \left. + \xi(1-\beta-x\gamma)(-1+\alpha+2\beta+2x\gamma)\right]\frac{q^2}{m^2} \\ G_2 &= -2\xi\alpha(-1+\alpha+2\beta+2x\gamma) \end{aligned} \tag{5.14}$$

Here account is taken of the photon mass $\lambda = m_{\text{ph}}^2/m^2$ in order to avoid infrared divergences in the vertex function. Let us write down expression (5.11) in the two limiting cases $m^2l^2 \ll 1$ and $q^2/m^2 \ll 1$.

The result is

$$F_2(q^2) = -\frac{\alpha}{2\pi}\left[1 - \frac{2}{3}v(1)m^2l^2\right] \tag{5.15}$$

and

$$F_1(q^2) = \frac{\alpha}{4\pi}\left\{\ln\frac{1}{m^2l^2} - 2\ln\frac{m^2}{m_{ph}^2} - v'(0) + \frac{7}{2} - \frac{8}{3}m^2l^2\right.$$
$$\left. + \left[v'(1) + v(1)\left(\ln m^2l^2 - \frac{35}{24}\right)\right]\right\} + \frac{\alpha}{2\pi}\frac{q^2}{m^2}$$
$$\times\left\{\frac{2}{3}\ln\frac{m}{m_{ph}} - \frac{3}{4} + \frac{2}{3}m^2l^2\left[v(1)\left(\ln m^2l^2 + \frac{1}{6}\right) + v'(1)\right]\right\} \quad (5.16)$$

where $\alpha = e^2/4\pi$ is the fine structure constant.

6. RESTRICTION ON THE "SIZE" OF THE ELECTRON

In the phenomenological nonlocal theory where we have introduced the form factor of the theory it should be understood that the parameter l may be interpreted as the "size" of extended objects, say electrons. There is definite interest in obtaining bounds on this parameter. Local quantum electrodynamics is the best tested theory in physics; so far no deviation has been found even at small distances. Tests of locality are usually performed by using very high precision experiments in physics. We apply here experimental data from measurements of anomalous magnetic moments (AMM) of leptons. A QED breakdown would imply that the electron (or the muon) has a finite size. The nonlocal contribution to the AMM is defined from the vertex function $\Lambda_\mu(q)$ in (5.11) containing the term with $\sigma_{\mu\nu}q_\nu$, i.e., (5.15).

At present, the experimental values (Particle Data Group, 1988) of the AMM of the electron and the muon are

$$\Delta\mu_{exp}^{(e)} = 1.001159652193 \pm 0.000000000010$$
$$\Delta\mu_{exp}^{(\mu)} = 1.001165923 \pm 0.000000008 \quad (6.1)$$

and are fully described by the local QED. Comparing the correction (5.15) with (5.17), one obtains bounds

$$l_e \leq 8.8 \times 10^{-15}\ \text{cm} \quad \text{for} \quad \Delta\mu_{exp}^{(e)} \quad (6.2)$$
$$l_\mu \leq 1.2 \times 10^{-15}\ \text{cm} \quad \text{for} \quad \Delta\mu_{exp}^{(\mu)}$$

This means that leptons are pointlike particles with radii smaller than 10^{-15} cm.

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